LETTERS TO THE EDITOR

COMMENTS ON "DISTORTION IMMUNITY OF MLS-DERIVED IMPULSE RESPONSE MEASUREMENTS"*

The above paper,[1] while a valuable contribution, fails to make an essential distinction between two different approaches to appraising distortion immunity, which may result in confusion in the minds of some readers. The authors conclude from their normalized error sequence of a third-order nonlinearity [see Fig. 15(c) and (d)] that "this behavior results in an error spike close to the linear impulse response, which cannot be removed by truncating the impulse response." While literally true, this statement requires more elaboration in order to be properly interpreted.

Indeed as Fig. 15(d) clearly shows, the cumulative error distribution suddenly jumps from 0% to over 40% for a third-order nonlinearity, and this is obviously due to the large initial spike visible in the error periodic-impulse-response (PIR) sequence of Fig. 15(c). What the authors fail to point out, however, is that such an initial spike will occur with *any* conventional measurement technique, including both time-delay spectrometry (TDS) and dual-channel fast Fourier transform (FFT). The latter uses statistical cross correlation between input and output, with random Gaussian noise excitation to obtain the impulse response. This spike occurs because any odd-order nonlinearity can always be approximated by a linear system, given an input excitation signal with known statistical properties *and* known amplitude.

The notion of a best-fit linear (or possibly higher order) approximation to a nonlinear system was first investigated by Norbert Wiener and later by Schetzen[2] and resulted in the Wiener theory of nonlinear systems, as opposed to the Volterra theory on which it is based and upon which the authors implicitly based their statement. Unlike the Volterra series, the Wiener series comprises terms (called G functionals) which are mutually orthogonal under Gaussian white-noise excitation, and this results in a least-squared error property not shared by the Volterra series.

To illustrate the difference between the Volterra and Wiener theories without reproducing a book full of theory, I'll use as an example a pure third-order nonlinearity which consists of a memoryless cubic operator preceded and followed by linear filters. The Volterra series of such a system consists of a single term, namely, the third-order Volterra kernel $h_3(\tau_1, \tau_2, \tau_3)$, convolved (in three dimensions) with the input signal to yield the output signal. This three-dimensional convolution operation term is more accurately called a third-order Volterra functional since it is actually a time function that depends on another time function, namely, the input. The three-dimensional Fourier transform of the Volterra kernel $H_3(f_1, f_2, f_3)$ is the Volterra nonlinear transfer function of this third-order system.

In contrast, the Wiener-series expansion of this same nonlinearity actually comprises the sum of two functionals (called G functionals in this case to distinguish them from Volterra functionals), a third-degree G functional *plus* a first-degree G functional (linear contribution). The reason for this difference is that the Wiener approach is one that seeks to minimize the mean-squared error between the input Gaussian noise and the output noise when the series is truncated to a lower order, regardless of how many or few G functionals are retained in the final series. Thus, for example, if we truncate this two-functional Wiener series to leave only the linear contribution, this remaining G functional represents the best-fit linear approximation to this hypothetical third-order nonlinear system. Truncating the single-functional Volterra series, in contrast, leaves us with the constant of zero, which is just an open circuit and is clearly not the best-fit linear approximation to a cubic nonlinearity. A constant actually is, incidentally, the best-fit linear approximation to any *even-order* nonlinearity. This explains why, when assessing distortion immunity, the distinction between Volterra and Wiener theories only becomes important when measuring systems containing odd-order nonlinearities. This is born out by the authors' Fig. 15(a) and (b), which shows the error PIR (a) and the cumulative error distribution (b) for a second-order nonlinearity. In this case there is no sudden jump in the error distribution, and there is also no clearly dominant spike in the error PIR sequence as there is in the case of a third-order nonlinearity (c).

Admittedly, this distinction between Volterra and Wiener theories was never made explicit in an earlier publication,[3] although it was alluded to in the section on MLS coherence by a statement regarding dual-channel FFT with Gaussian excitation: "In that case, the transfer function can be identified as

$$H_{xy}(f) = \frac{S_{xy}(f)}{S_{xx}(f)} \tag{38}$$

which Wiener originally proved and which represents the best possible estimate of the system transfer function

* Manuscript received 1993 May 28.

[1] C. Dunn and M. O. Hawksford, *J. Audio Eng. Soc.*, vol. 41, pp. 314–335 (1993 May).

[2] M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems* (Robert E. Krieger, Malabar, FL, 1989).

[3] D. D. Rife and J. Vanderkooy, "Transfer-Function Measurement with Maximum-Length Sequences," *J. Audio Eng. Soc.*, vol. 37, pp. 419–444 (1989 June).

in the sense that it minimizes the mean-square error between $y(t)$ and $x(t) * h_{xy}(t)$. In other words, if the actual system is noisy or exhibits nonlinearities, $H_{xy}(f)$ is the best-fit linear approximation to the actual system behavior, and the coherence function, defined as

$$\tau^2_{xy}(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)} \quad (39)$$

expresses the degree to which the input and output signals are linearly related by $H_{xx}(f)$." In retrospect I might have added that $h_{xy}(t)$ is the first-order *Wiener kernel* (as opposed to the first-order Volterra kernel) and furthermore that the MLS method yields $h_{xy}(t)$ directly as the initial spike in the PIR using a relatively short time window to exclude the artifacts in the PIR tail, which represent the contributions of the higher order Wiener kernels. The Fourier transform of this first-order Wiener kernel, namely, $H_{xy}(f)$, is, of course, the optimum linear transfer function of the nonlinear system when subject to Gaussian noise excitation.

It is easy to show that TDS, like MLS, also does not completely reject odd-order nonlinearity in the Volterra-series sense implicitly used by the authors. In TDS a swept sine wave is the stimulus and a tracking filter at the output rejects any harmonics generated by the nonlinearities, provided the sweep rate is sufficiently low. But consider that a pure sine wave applied to any third-order nonlinearity will generally result in *two* output frequencies,[2] namely, the third harmonic which is rejected by the tracking filter *plus* the fundamental frequency which cannot be rejected no matter how narrow one makes the tracking filter's bandwidth. Indeed, in TDS the fundamental frequency at the output is always interpreted as the linear component of the overall response. Thus TDS, MLS, as well as dual-channel FFT methods will all show a spike near time zero in their respective error impulse responses when measuring systems containing any odd-order nonlinearity.

As pointed out by the authors, inverse repeat sequences do reject all even-order kernels of the Volterra-series expansion. This effect can be achieved using normal MLS methods as well. Simply perform a normal MLS measurement and denote the resulting PIR sequence as $h_+(n)$, then repeat the measurement, but this time with the polarity of the driving MLS reversed, and denote this result by $h_-(n)$. Two derivative PIR sequences can now be formed, $h_{\text{even}}(n)$, which rejects all the odd-order Volterra kernels (including the linear kernel), and $h_{\text{odd}}(n)$, which rejects all the even-order kernels. These two sequences are obtained as

$$h_{\text{even}}(n) = \frac{h_+(n) + h_-(n)}{2} \quad (1)$$

$$h_{\text{odd}}(n) = \frac{h_+(n) - h_-(n)}{2} . \quad (2)$$

See Schetzen[2] (fig. 5.4-1) for a justification of these expressions.

In conclusion, if one uses the Wiener-series expansion of nonlinear systems as the basis for appraising distortion immunity, a filtered MLS which exhibits a nearly Gaussian probability density function (PDF) is a nearly ideal stimulus since the Wiener theory assumes a Gaussian input as its starting point. Therefore in the Wiener-series sense (meaning in this context the best-fit *linear* approximation to a given nonlinear system), MLS methods do in fact completely reject all nonlinear distortion of finite order in the limit as the MLS period goes to infinity provided the analysis window is finite and the binary MLS is prefiltered to yield a nearly Gaussian PDF. In other words, when this limit is reached, one has obtained a transfer function which is the best possible linear approximation to the actual system behavior in the least-squared error sense.

If, in contrast, one uses the Volterra-series expansion as the basis for appraising distortion immunity, then indeed the authors' implicit conclusion that MLS methods lack complete immunity to odd-order nonlinearities is quite correct. In that case, however, it is only fair to point out that this same conclusion applies equally to other commonly available measurement techniques, including TDS as well as dual-channel FFT methods, which similarly fail to completely reject the odd-order nonlinearities in the Volterra-series sense.

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Authors' Reply[4]

We would like to thank Mr. Rife for his interest in our paper[1] and make a couple of comments concerning the points raised. Rife points out that the uneven cumulative energy distribution for the third-order nonlinearity with memory that occurs in maximum-length sequence (MLS) measurements [see Fig. 15(d)] also occurs in time-delay spectrometry (TDS) and other measurement strategies. While this is true, the intention in including this particular simulation result in the paper was not to criticize the performance of MLS with respect to other possible measurement regimes, but to investigate the nature of the MLS error distribution when the nonlinearity includes memory. We showed that memory in even-order nonlinearities tended to smooth the error distributions [compare Figs. 11(b) and 15(b)], but for odd-order-nonlinearity with memory the "gain error" is no longer coincident in time with the linear impulse response—hence it cannot be considered to be a linear error, and an uneven error distribution results [compare Figs. 11(d) and 15(d)]. Such an uneven error distribution implies that, for odd-order nonlinearity with memory, MLS distortion immunity cannot be indefinitely enhanced by using longer and longer measurement periods and truncating the recovered impulse response.

In our paper,[1] we showed how inverse-repeat se-

[4] Manuscript received 1993 November 23.

quences (IRS) exhibit complete immunity to even-order nonlinearity, and Rife shows how this behavior can also be obtained with MLS by performing two measurements where the second excitation signal is an inverted version of the first, and subtracting the resultant impulse responses from one another. Such a process would require two cross-correlation operations, and a faster implementation requiring only a single cross correlation would be to subtract the second MLS period measured at the output of the system from the first MLS period *before* cross correlation (we shall term this method the inverted MLS approach). In fact, both the IRS and the inverted-MLS methods require similar measurement and cross-correlation times and have the same maximum measurable impulse lengths given a hardware memory limit, but IRS has a slight advantage in terms of *odd*-order distortion immunity.

Consider an MLS of period L samples. If two measurements were made of a system where the second measurement uses an inverted excitation, then the minimum possible measurement time is $4L$. This is because for the noninverted measurement, L samples must precede data capture so that the system can settle to steady-state conditions, while a similar settling period of L samples must precede the inverted measurement. An inverse-repeat sequence generated from an MLS of period L results in an IRS of length $2L$, hence the minimum measurement time for IRS is also $4L$.

Blome[5] has recently shown how a $2L$-point IRS cross correlation can be executed with a single $(L + 1)$-point fast Hadamard transform (FHT). Such a cross-correlation procedure would only recover the first L samples of the true $2L$-point IRS cross correlation, but since the second L samples of the IRS cross correlation are simply an inverted version of the first L samples, this is of no consequence. Blome's finding suggests that cross-correlation times for the inverted-MLS and IRS approaches are equal.

In our paper[1] we suggested that, compared to MLS, IRS suffers from a halving of maximum measurable impulse length, given a hardware memory limit. However, using Blome's IRS cross-correlation routine with a minor modification, a $2L$-point IRS measurement can recover an L-point impulse response using only L memory locations for data capture. Hence maximum recoverable impulse lengths are identical for MLS, inverted-MLS, and IRS measurement approaches.

Although the inverted-MLS method exhibits complete immunity to even-order nonlinearity, odd-order distortion immunity is identical to that obtained with a standard MLS measurement. However, we showed[1] that for systems where the bandwidth is significantly lower than the sampling frequency of the measurement system, IRS has some odd-order distortion immunity advantage over MLS. The simulations included in the paper (Table 1, p. 320) of a 1-kHz FIR low-pass filter measured at a sampling rate of 44.1 kHz indicated an odd-order distortion immunity advantage for IRS that increased with the order of nonlinearity, being approximately 1 dB for third-order nonlinearity and 5 dB for seventh-order distortion. This advantage is a result of the superior odd-order autocorrelation characteristics of the IRS excitation signal.

One final point not raised in our paper[1] is that of measuring polarity in the test system. For an MLS measurement, if the polarity of the excitation is known, then the polarity of the periodic impulse response recovered from cross correlation reveals whether the test system is inverting or noninverting. However, for both inverted-MLS and IRS measurements the polarity of the test system is not revealed by the polarity of the recovered impulse response unless some form of synchronization is used to time-align data capture with the excitation signal.

To summarize, we have noted that, for a given recovered impulse response length, inverted-MLS and IRS techniques possess identical measurement and cross-correlation times. Given a hardware memory limit, the two techniques have the same maximum recoverable impulse response length. Both inverted-MLS and IRS methods exhibit complete immunity to even-order nonlinearity, but IRS has a small advantage in terms of odd-order distortion immunity.

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Further Comments[6]

I thank Dr. Dunn for his reply to my comments. I feel, however, that my main point still needs to be reiterated. The quite correct conclusion drawn by Dunn and Hawksford, namely, that MLS measurements of systems containing odd-order nonlinearities fail to completely reject those nonlinearities regardless of the sequence length, assumes a Volterra-series-based definition of the error impulse response. If one changes to a Wiener-series-based definition, this conclusion no longer holds. To be more precise, let's define the error impulse response first in terms of the Volterra series and then in terms of the Wiener series. The definition in terms of the Volterra series is: 1) form the Volterra-series expansion of the nonlinear system to be measured, 2) drop all terms higher than first order, 3) subtract the remaining first-order (linear) term from the measured MLS response, and then 4) after cross correlation, what remains is the residual (unrejected) nonlinear distortion. The definition in terms of the Wiener series is very similar: 1) form the Wiener-series expansion of the nonlinear system to be measured, 2) drop all terms higher than first order, 3) subtract the remaining first-order (linear) term from the measured MLS response, then 4) after cross correlation, what remains is the residual nonlinear distortion. Note here that a nonlinear system with mem-

[5] C. A. Blome, private communication (1993).

[6] Manuscript received 1993 December 6.

ory as described by Eq. (14)[1] does indeed exhibit a delayed "gain-error" spike in the Volterra-series-based error impulse response, which must therefore be regarded as a nonlinear artifact. Under the Wiener-series definition, however, even for nonlinear systems with arbitrary amounts of memory, this is no longer the case. The first-order term of the Wiener series expansion easily accommodates such a delayed gain error or, in fact, any effect that can be described by a purely linear system, and so this delayed gain-error spike will no longer appear in a Wiener-series-based error impulse response. In this sense the Wiener series is a more realistic description than the Volterra series because the former, unlike the latter, includes in its higher order terms *only* those effects that absolutely cannot be described by a purely linear system, no matter how complex.

I totally agree with the authors' statements regarding inverted-MLS versus IRS measurements. There is, however, a significant advantage to the inverted-MLS procedure if the object is to measure distortion rather than to merely reject it. By computing the odd- and even-order PIR sequences it becomes possible to compute the odd- and even-order coherence functions (or the related incoherence functions), whereas with the IRS method the even-order Volterra terms (as well as the even-order Wiener terms) are totally rejected from the start and can never be recovered, not unlike absolute polarity.

Note also that to avoid confusion and to be very precise, the term "coherence function" should actually read "Wiener coherence function" since coherence measurements using traditional dual-channel analyzers implicitly assume a Wiener-series definition of nonlinearity. For example, if one were to actually measure the authors' example system [the one containing the delayed gain-error nonlinearity described by Eq. (14)][1] with a standard dual-channel instrument, it would show a lower incoherence (that is, lower distortion or higher coherence) than would have been predicted based on the Volterra-series expansion of that system, that is, by the ratio of the power in the sum of all its higher order (presumably, distortion-containing) terms to its total power.

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Authors' Reply to Further Comments[7]

We would once again like to thank Mr. Rife for his interest in our paper.[1] In his 1993 November communication on this subject, Rife raised several points:

1) If a Wiener approach to nonlinearity is adopted, then "MLS measurements do in fact completely reject all nonlinear distortion of finite order in the limit as the MLS period goes to infinity." This statement contradicts the authors' finding,[1] where we showed that for the specific case of odd-order nonlinearity *with memory*, MLS distortion immunity cannot be indefinitely increased by truncating the impulse response recovered from cross correlation.

2) If a Volterra approach to nonlinearity is adopted, then "MLS methods lack complete immunity to odd-order nonlinearities. . . . In that case, however, it is only fair to point out that this same conclusion applies equally well to other commonly available measurement techniques. . . ."

3) Although linear transfer-function measurements performed using inverse-repeat sequences exhibit complete immunity to all even-order nonlinearity, this effect can also be obtained by performing two MLS measurements, where the second MLS excitation is an inverted version of the first, and subtracting the resultant impulse responses from one another. We term this method the "inverted-MLS" approach.

In the authors' reply to this first communication we dealt with the first two of these points, albeit briefly, and without reference to the alternative Wiener and Volterra theories of nonlinearity, by discussing MLS distortion immunity for odd-order nonlinearity with memory. We will consider this matter in more detail here. As for the last point, although we acknowledged that an inverted-MLS approach to linear transfer-function measurement does indeed exhibit complete immunity to even-order nonlinearity, we suggested that, from a distortion *immunity* perspective, inverse-repeat sequences possess superior characteristics in terms of *odd*-order distortion immunity.

In the most recent communication Rife again spends some time describing the differences between Volterra and Wiener theories of nonlinear systems. As for inverted-MLS techniques, Rife further points out that such measurements are capable of distinguishing between odd- and even-order nonlinearity, unlike inverse-repeat sequences where all even-order nonlinearity is completely rejected in the PIR recovered from cross correlation. While this is true, the intent of the original paper[1] was to study distortion and noise immunity in linear transfer-function measurements rather than the ability to distinguish between different types of nonlinearity. There is also the question of how valuable a measurement of total even-order or total odd-order distortion is, since although it is well known that, from a psychoacoustic perspective, a given amount of odd-order nonlinearity tends to be more annoying than the same amount of even-order distortion, such measurements cannot (yet) accurately distinguish between low-order and high-order distortion. Hence an inverted-MLS measurement of even-order nonlinearity could not distinguish between second-order and sixth-order distortion, although the latter would be far more annoying in an audio sense.

We now progress to a discussion of the main points raised by Rife, concerning the differences between Wiener and Volterra theories of nonlinearity, and how they relate to distortion immunity in MLS measurements. We should point out at this stage that the following arguments do not change any of the findings in our paper,[1] which we feel remain essentially correct, but should be

[7] Manuscript received 1994 January 12.

seen as an addendum which may aid in the understanding of MLS distortion immunity.

We completely agree that, from a Wiener perspective of nonlinearity, MLS measurements completely reject all nonlinearity given a long enough sequence period. However, we do not agree that the Wiener approach of analyzing nonlinearity is the most appropriate when it comes to linear transfer-function measurement in audio systems. This is not to say that we favor a Volterra description of nonlinear distortion when analyzing such measurements. In fact, we feel a flexible position which considers both approaches to be most appropriate.

If a Volterra description of nonlinearity is adopted in linear transfer-function measurement of weakly nonlinear systems, then *any* component of the recovered impulse response which is due to nonlinearity is classed as a nonlinear artifact, even if it is coincident with and of the same shape as the linear impulse response—the type of error which, for example, is caused by memoryless, odd-order nonlinearity in an MLS measurement. A Volterra definition of distortion is in this latter case unattractive since it includes errors due to nonlinearity which do not change the *shape* of the recovered impulse response, only its relative magnitude—a "gain change" which is not of great significance in loudspeaker measurement, for example. (This concept is discussed in more detail in our paper.[1]) However, if the test system is subject to odd-order distortion *with memory*, then an MLS measurement of the system will contain a relatively large error spike in the recovered PIR that is not coincident with the linear impulse response. Such a delayed error spike must change the *shape* of the magnitude response which is obtained by Fourier transforming the PIR, and hence must be of significance when considering the distortion immunity of the measurement. In this case the Volterra definition of nonlinearity *is* appropriate, unlike the Wiener description, which considers any such first-degree G functional to be part of the linear system response.

We feel that the following simulation examples aid an understanding of the differences of odd-order nonlinearity with and without memory. First consider the PIR shown in Fig. 1 obtained from a 4095-point MLS measurement of the FIR 1-kHz low-pass filter described in our paper,[1] where the measurement has been corrupted by third-order distortion at a relative level of 0 dB. Referring to our Eq. (2), this nonlinearity can be written as

$$d\{x_f(n)\} = [x_f(n)]^3 \,. \tag{1}$$

Note that the error due to nonlinearity is spread approximately evenly in the tail of the impulse response. Fig. 2 shows the first 256 samples of this corrupted impulse response, where the peak of the impulse response occurs at approximately 110 samples. Fig. 3 illustrates the first 256 samples of the impulse error due to nonlinearity, where a relatively large error peak is coincident with the linear impulse response at 110 samples. In fact this large peak is a scaled version of the linear impulse response, hence it simply affects a gain error in the measurement and does not corrupt the shape of the derived transfer function. We described this gain error in Eq. (6),[1] and removing the gain-error component from the error signal results in a normalized error sequence which is essentially evenly distributed across the measurement period. The first 256 samples of the normalized error sequence in this example are shown in Fig. 4, characterized by the absence of any peak at 110 samples.

Now consider the effects of nonlinearity in the frequency domain. Fig. 5 shows the magnitude response of the FIR filter used in the simulation, where the passband is essentially flat. Fig. 6 shows the magnitude response obtained by Fourier transforming the whole im-

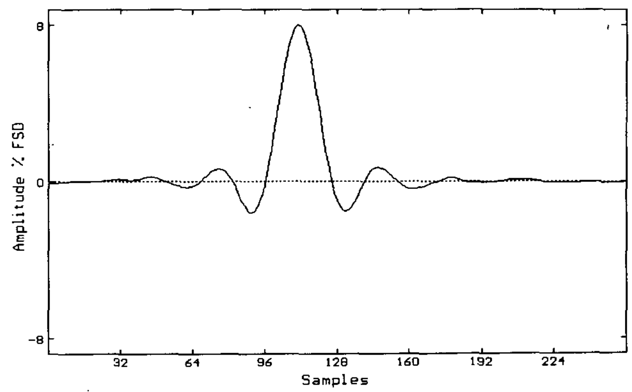

Fig. 2. PIR.

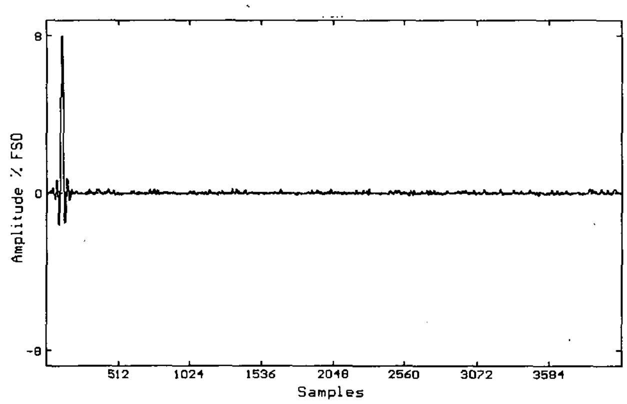

Fig. 1. PIR.

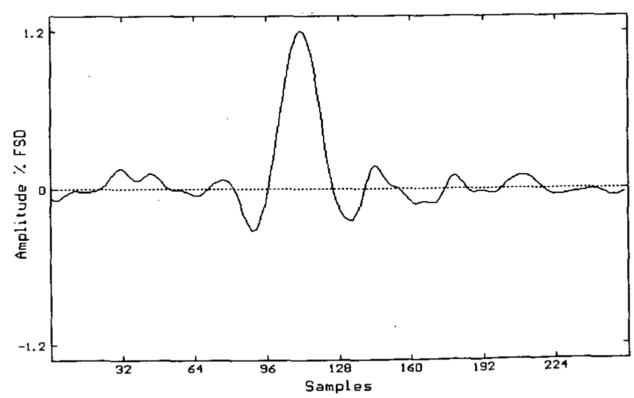

Fig. 3. PIR error.

pulse sequence shown in Fig. 1; the nonlinearity has had the effect of raising the observed passband gain of the filter, and also of superimposing a "noise" upon the measured response. Note, however, that the *shape* of the linear filter is preserved in the corrupted measurement. Fig. 7 shows a second magnitude response obtained from the same impulse response shown in Fig. 1, but truncated to 256 samples. Although the smaller number of samples has reduced the frequency resolution somewhat, the truncation has also reduced the noiselike error due to nonlinearity—behavior that is commensurate with the idea that MLS distortion immunity increases for a given analysis window length as the MLS period increases. The gain change within the filter passband due to the nonlinearity is also evident, and if this is considered to be a benign error, since it does not change the shape of the recovered linear transfer function, then this example of MLS distortion immunity with memoryless nonlinearity clearly indicates that a Wiener approach to classifying nonlinear errors is appropriate.

We now consider a second simulation example, where third-order distortion with memory corrupts the measurement. Each term input to the nonlinearity is delayed by 40 samples, that is,

$$d\{x_f(n)\} = [x_f(n - 40)]^3 . \quad (2)$$

Fig. 8 shows the first 256 samples of the PIR obtained from a 4095-point MLS measurement of the same FIR filter used in the first simulation example, while Fig. 9

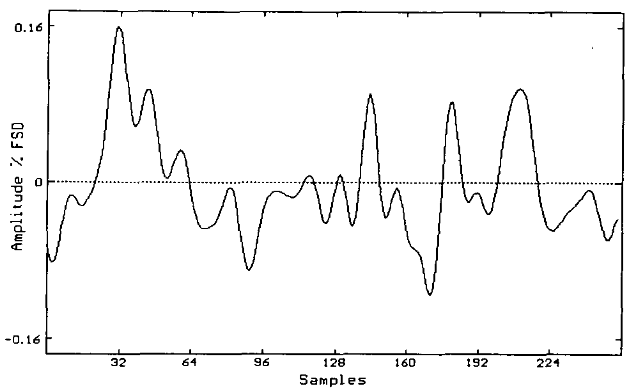

Fig. 4. Normalized PIR error.

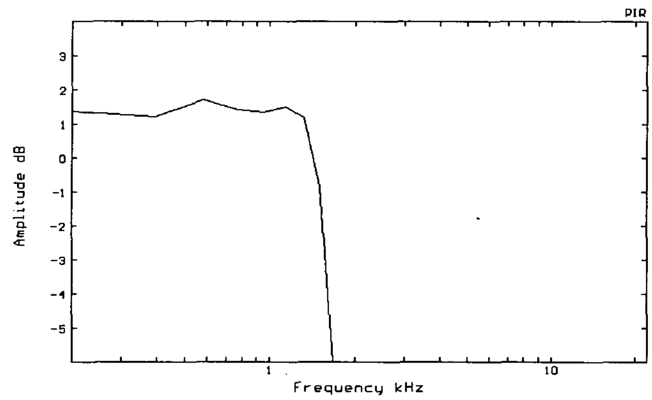

Fig. 7. 256-point FFT.

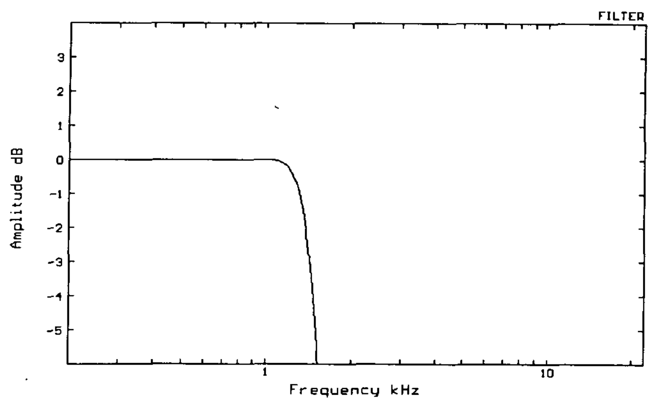

Fig. 5. 4096-point FFT.

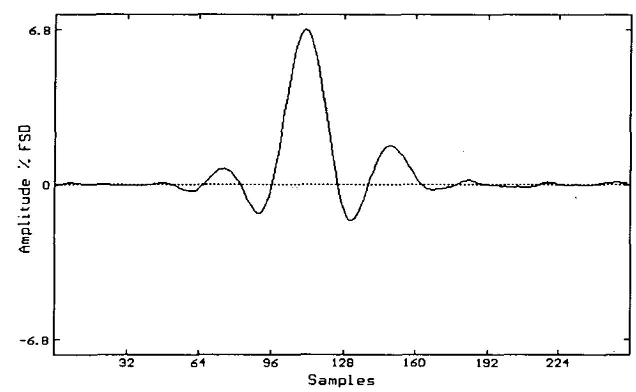

Fig. 8. PIR.

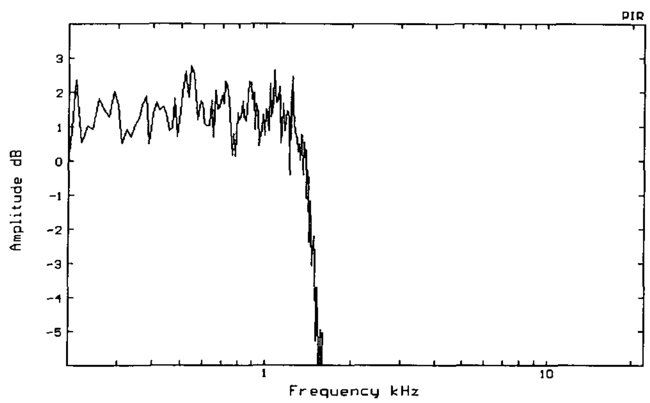

Fig. 6. 4096-point FFT.

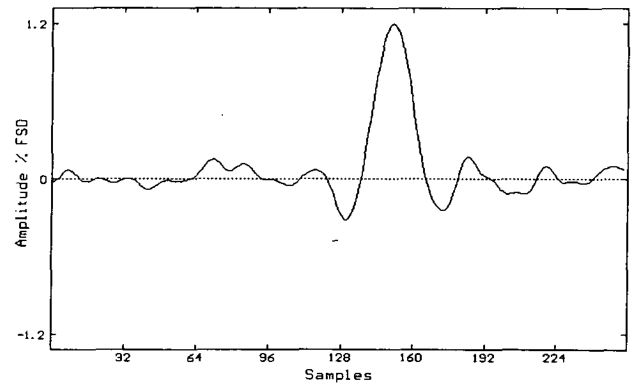

Fig. 9. PIR error.

shows the error in this measurement due to nonlinearity. The error peak is clearly delayed by 40 samples relative to the peak of the linear impulse response, and this delay is a direct consequence of memory within the nonlinearity. Because the linear and error peaks are noncoincident, then normalizing for gain change results in very little change in the time-domain error signal (Fig. 10). The delayed error peak due to nonlinearity in this example results in a change in the shape of the linear transfer function obtained from the PIR, Fig. 11 showing a peak in the magnitude response at approximately 1 kHz and a trough at 500 Hz, as well as a superimposed noiselike error also observed in the first (memoryless nonlinearity) example. Importantly, truncating the PIR to 256 samples (Fig. 12) shows a reduction in noise due to nonlinearity, but the shape of the magnitude response remains corrupted within the filter passband because the relatively large delayed error peak has not fallen outside the analysis window. In fact, in this example no truncation scheme can effectively remove the magnitude response corruption due to nonlinearity without also truncating the linear component of the PIR recovered from cross correlation. Although this example is somewhat contrived in that the nonlinearity has been selected to result in a large error spike which occurs in the same region as the linear component of the PIR, it does illustrate the fact that MLS distortion immunity cannot be indefinitely enhanced by increasing the MLS measurement period when the nonlinearity is of odd order and contains some memory. Hence the Wiener approach to nonlinear error analysis in MLS measurements is in this case invalid, and in fact the Volterra approach is more attractive here.

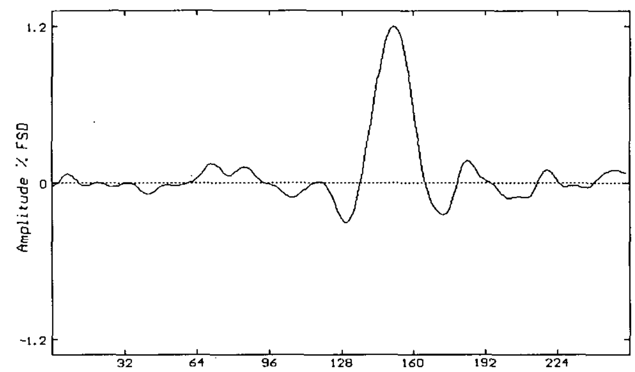

Fig. 10. Normalized PIR error.

Rife pointed out that if a Volterra definition of nonlinearity is adopted, then MLS methods lack complete immunity to odd-order nonlinearities, but that this conclusion applies equally well to other commonly available measurement techniques. While we agree with this statement for memoryless nonlinearity, we believe it is not true for odd-order distortion with memory. Consider, for example, a TDS measurement with a very fast sweep rate—here *all* error terms due to nonlinearity with memory will be at different instantaneous frequencies relative to the linear term, and if a filtering operation with good frequency discrimination is employed, then such errors can be distinguished from the linear term. Here the essential difference between TDS and MLS techniques is that the instantaneous frequency of a TDS stimulus changes with time, while for MLS all frequencies are present all of the time.

We believe that it is easy to misunderstand these ideas if one sticks too rigidly to a theoretical concept of distortion immunity. Rife states that ". . . in a Wiener-series sense (meaning in this context the best-fit *linear* approximation to a given nonlinear system), MLS methods do in fact reject all nonlinear distortion of finite order in the limit as the MLS period goes to infinity. . . ." Although a truncated MLS measurement, which includes delayed-error spikes due to odd-order distortion with memory, may indeed result in the best-fit linear *approximation* to that nonlinear system for a single measurement, it is wrong to conclude that the truncated analysis window contains only linear components, even in the limit as the measurement period approaches infinity. It is easy to see that delayed error spikes due to nonlinearity cannot be linear components of the PIR if one considers what happens for, say, a cubic nonlinearity with memory as the amplitude of the stimulus exciting the nonlinearity decreases. This is shown in a third example (Fig. 13) using the same filter and nonlinearity as in the second example, but where the MLS amplitude input to the filter has decreased by approximately 6 dB, and the PIR recovered from cross correlation has been scaled to take into account this change in stimulus amplitude. Note

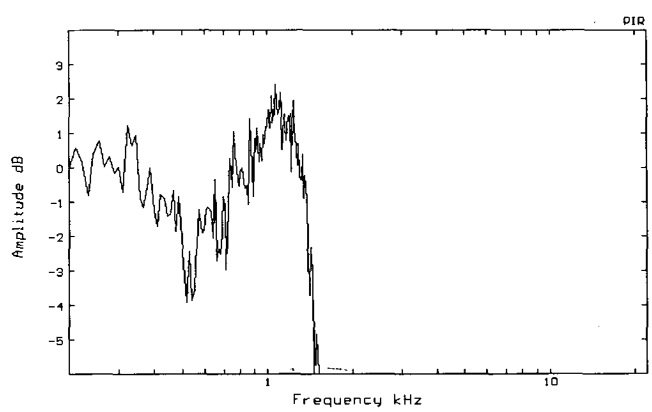

Fig. 11. 4096-point FFT.

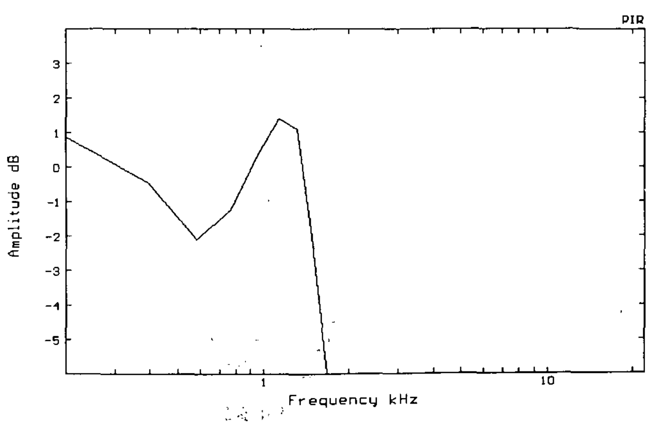

Fig. 12. 256-point FFT.

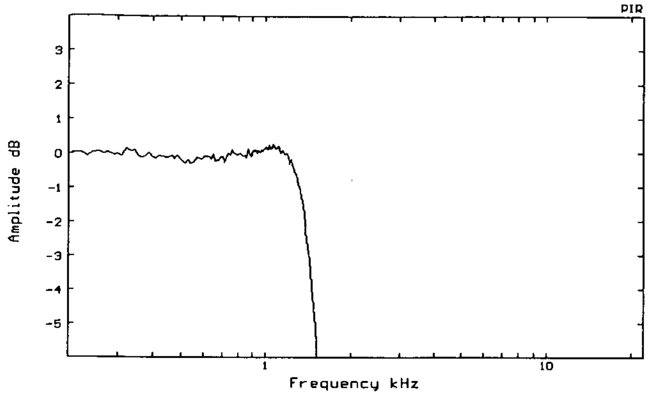

Fig. 13. 4096-point FFT.

that the magnitude-response shape has again been corrupted by the delayed error spike, but the degree of corruption is not as severe as in Fig. 11. In fact, with this particular example the shape of the recovered PIR changes as the stimulus amplitude changes, and this behavior is maintained even if the recovered PIR is truncated and the period of the MLS tends toward infinity. However, this behavior is inconsistent with the notion that a measurement which exhibits complete distortion immunity results in a linear transfer function whose shape is invariant with stimulus amplitude.

To summarize, for memoryless odd-order nonlinearity an MLS measurement recovers a PIR which contains an error spike of the same shape as and coincident with the linear component of the impulse response. Therefore this error merely results in a gain change in the measurement. Hence for all even-order and memoryless odd-order nonlinearities, a Wiener description of nonlinearity is appropriate—which implies that an MLS measurement exhibits complete immunity to these types of distortion as the MLS measurement period approaches infinity. However, for odd-order nonlinearity with memory, the error spike is delayed relative to the linear response, and hence changes the shape of the linear-transfer function recovered from the measurement. Under these circumstances a Volterra approach seems more appropriate in describing nonlinearity, in which case MLS distortion immunity cannot be indefinitely increased by increasing the measurement period.

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COMMENTS ON "HORN MODELING WITH CONICAL AND CYLINDRICAL TRANSMISSION-LINE ELEMENTS"[8]

In the above paper,[9] Mr. Mapes-Riordan used 100 000 conical transmission-line elements to model a conical horn. We do not understand why more than one conical element is needed to model a conical horn. The conclusion of the paper stated that conical elements converged more quickly than cylindrical elements when used to model general horn contours. The argument for the superiority of the conical elements in modeling general horn contours is not justified by investigating the convergence of conical elements in modeling a conical horn.

Conical sections are presented as "spherical-wave" elements and cylindrical sections are presented as "plane-wave" elements. It is true that conical elements may be used for both spherical-wave and plane-wave modeling, but no number of elements will cause a plane-wave field to converge to a spherical-wave field. It does not seem relevant to compare the two types of elements unless the theories are being compared with measured data to verify that the experimental results are best predicted assuming plane wavefronts or spherical wavefronts.

Useful references related to the topics contained in Mr. Mapes-Riordan's paper can be found in the literature. The topic of transmission-line modeling of horn loudspeakers was studied by McLean et al.,[10] who demonstrate the utility of transmission-line modeling for predicting the acoustic impedance at the throat of horns. Thermoviscous losses are included, and the numerical results are compared with laboratory measurements. The original idea of using lumped transmission lines for modeling acoustic horns should be credited to A. G. Webster, who also presented explicit expressions for the transmission matrices of both cylindrical and conical horns (although in an antiquated notation) in his paper on acoustic impedance.[11] Also, a method that may prove useful for studying the convergence of various horn elements is presented by Bonder,[12] who uses the approach to find the number of resonances in a horn.

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Author's Reply[13]

I will address two issues brought up by John T. Post and Elmer L. Hixson in their response to my paper on horn modeling. First, they question why more than one conical element is needed to model a conical horn. Second, they interpret part of my conclusion to mean that

[8] Manuscript received 1994 January 3.

[9] D. Mapes-Riordan, *J. Audio Eng. Soc.*, vol. 41, pp. 471–484 (1993 June).

[10] J. S. McLean, J. T. Post, and E. L. Hixson, "A Theoretical and Experimental Investigation of the Throat Impedance Characteristics of Constant Directivity Horns," *J. Acoust. Soc. Am.*, vol. 92, pp. 2509–2526 (1992 Nov.).

[11] A. G. Webster, "Acoustical Impedance, and the Theory of Horns and of the Phonograph," *Proc. Nat. Acad. Sci.*, vol. 5, pp. 275–282 (1919), read in 1914 December at the meeting of the American Physical Society at Philadelphia. Reprinted in R. B. Lindsay, Ed., *Benchmark Papers in Acoustics*, vol. 4, *Physical Acoustics* (Dowden, Hutchinson & Ross, Stroudsburg, Pa., 1973).

[12] L. J. Bonder, "The *n*-Tube Formula and Some of Its Consequences," *Acustica*, vol. 52, pp. 216–226 (1983).

[13] Manuscript received 1994 January 26.